Contact Dynamics in a Gently Vibrated Granular Pile

Alexandre Kabla and Georges Debrégeas*
LFO-Collège de France, CNRS UMR 7125, Paris, France
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We use multispeckle diffusive wave spectroscopy to probe the micron-scale dynamics of a water-saturated granular pile submitted to discrete gentle taps. The typical time scale between plastic events is found to increase dramatically with the number of applied taps. Furthermore, this microscopic dynamics weakly depends on the solid fraction of the sample. This process is largely analogous to the aging phenomenon observed in thermal glassy systems. We propose a heuristic model where this slowing-down mechanism is associated with a slow evolution of the distribution of the contact forces between particles. This model accounts for the main features of the observed dynamics.

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Internal contact forces in dense granular systems are very inhomogeneous [1,2], even for crystalline assemblies [3]. The stress at a given contact can change macroscopically following a relative displacement of the two particles on the order of microns. Hence, large modifications in the contact forces field can result from minute deformations of the pile. This phenomenon is crucial in understanding the catastrophic yielding occurring in granular systems submitted to a slowly varying stress (avalanches [4], shear bands in triaxial tests [5]). It also explains why the sound transmission through a granular sample can be strongly affected by very small deformations [6]. To probe the evolution of the internal stress field, one needs to measure forces directly [7,8] which is difficult in 3D. In this Letter, we propose a different approach: we use multispeckle diffusive wave spectroscopy (MSDWS) to measure particle displacements on micron scales in a pile submitted to gentle discrete taps. These vibrations are too weak to induce large-scale rearrangements which would eventually lead to a compaction of the granular system [9–11]. We can therefore evaluate the microdynamics of the contacts without significantly perturbing the packing structure.

We use glass beads of diameter 45 ± 2 μm, contained in a glass cell (30 mm $\times$ 10 mm $\times$ 2 mm). To reduce electrostatic forces and avoid any capillary attraction due to moisture, the granular system is fully saturated with pure water. During the experiment, the mean packing fraction ϕ is obtained by measuring the position of the upper surface of the pile with a charge-coupled device (CCD) camera. Although a systematic error of 2% cannot be avoided, this allows us to detect relative changes in ϕ as small as 0.01%. To produce motion in the pile, we use a piezoelectric actuator on which the cell is rigidly mounted. Vertical vibrations of precisely controlled amplitude, shape, and durations can thus be applied to the granular column. In the present experiment we focus on a single type of mechanical excitation, later referred to as a "tap," which consists of a train of square wave vibrations of frequency 1 kHz and duration 100 ms. Different applied voltages are used, yielding various vertical amplitudes ranging from 50 to 300 nm.

In a standard experimental run, the pile is prepared by turning the cell upside down and then allowing the particles to sediment for half an hour. This procedure yields reproducible structures of low volume fraction. The pile is then submitted to high amplitude taps (of vertical amplitude 300 nm) until it reaches a prescribed packing fraction ϕ_s. This first step is a way to obtain granular samples of given packing fractions with essentially the same preparation history. We then start probing the dynamics of contacts by submitting the cell to very gentle taps of amplitude 50 nm (Fig. 1). During the compaction stage, the evolution of the packing fraction ϕ with the number of taps is consistent with previous experimental results on dry granular systems [9–11]. It should be noted however that the packing fraction we reach is well above the close

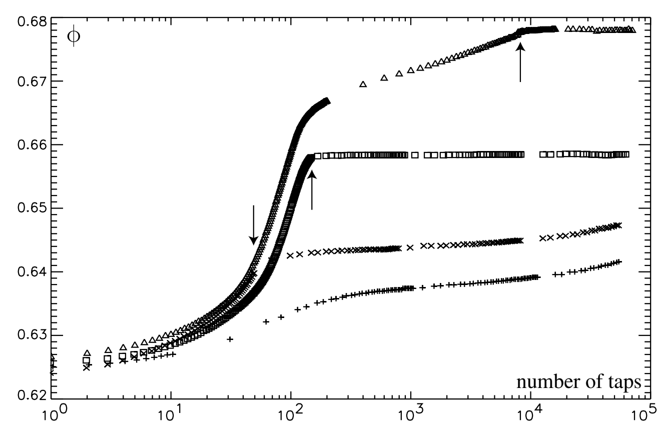

FIG. 1. Evolution of the packing fraction for four experimental runs. Each run consists of a first step in which high amplitude taps allow rapid compaction of the sample, followed by a sequence of gentle vibrations, during which the internal dynamics is probed. The arrows indicate the change in tapping intensity, which occurs after (+) 0, ($\times$) 50, ($\square$) 150, ($\triangle$) 8000 high impulsion pulses. There is a systematic error of 2% on the measurements of the packing fraction.

packing limit expected for a fully disordered pile ($\simeq$0.64). This indicates that crystallization does occur in our system under strong vibration. In contrast, the low intensity vibrations do not induce significant further evolution of the packing fraction except for initially very loose packs.

To probe the microscopic dynamics induced by these gentle taps, we use MSDWS [12,13]. This technique, which allows one to resolve submicron displacements, has been successfully applied to granular dynamics by several groups [14,15]. The sample is illuminated with a He-Ne laser beam at a depth of 2 cm below the upper surface of the pile (1 cm over the bottom). Photons are multiply scattered by the particles [16] and form a speckle pattern on the opposite cell wall which we record with a CCD camera. In the absence of vibration, the speckle image does not change in time as temperature is insignificant for such large objects. In contrast, the taps induce some irreversible particle displacements which modify the speckle image. To quantify the internal dynamics, we measure the intensity correlation of speckle images, taken between taps, as a function of the number of taps t that separate them. This function generally depends on the total number of small amplitude taps t_w that have been performed. We therefore calculate the two-times correlation function $g(t_w, t)$:

$$g(t_w, t) = \frac{\langle I(t_w + t)I(t_w)\rangle_{\rm spkl} - \langle I(t_w)\rangle^2_{\rm spkl}}{\langle I(t_w)^2\rangle_{\rm spkl} - \langle I(t_w)\rangle^2_{\rm spkl}}. \qquad (1)$$

In this expression, $\langle\,\rangle_{\rm spkl}$ denotes the average over different speckles. MSDWS thus allows one to rapidly access relaxation time by substituting time with space averaging and is therefore well suited to the study of nonstationary dynamical systems.

Figure 2 shows three correlation functions obtained with the same sandpile at different values of t_w. These functions are well fitted by stretched exponentials: $g(t_w, t) = \exp(-(t/\tau(t_w))^{\alpha(t_w)})$.

For different packing fractions, we follow the evolution of the dynamics by monitoring $\tau(t_w)$ and $\alpha(t_w)$ as a function of the total number of gentle pulses t_w. We find that the exponent α is roughly constant ($\simeq 0.8 \pm 0.2$) and independent of the packing fraction ϕ_s. In contrast, the time $\tau(t_w)$ increases by five decades over the range of t_w explored, as shown in Fig. 3. This result demonstrates that the response of a granular system to small perturbations is strongly dependent on the history of its preparation (the number of applied taps t_w) and rather insensitive to the packing fraction. Conversely, the dynamics can be entirely reset by submitting the system to a few taps of large intensity (such as those used for compacting the sample). A careful examination of the $\tau(t_w)$ curve also reveals large fluctuations in the internal dynamics, especially in looser packs in which the packing fraction slowly evolves. During certain periods of time, the dynamics is restarted as shown by a sudden decrease of $\tau(t_w)$. This may correspond to catastrophic failures of the pack structure, which compete against a global reinforcement of the granular contacts.

The dynamical arrest observed here is strongly reminiscent of the aging behavior recently exhibited in various glassy colloidal systems [18–20]. In these materials, the longest (α)-relaxation time is found to grow as a power law of the time since the system was left to rest after rapid shearing. Here, the large fluctuations of $\tau(t_w)$ do not allow one to definitely claim a power-law behavior, especially for low density samples, but the overall evolution of $\tau(t_w)$ is compatible with this standard result. This analogy is surprising considering the differences in the microscopic processes underlying the dynamics in both systems: in glassy liquids, stress relaxation occurs by thermally activated rearrangements of the structure. In granular materials, temperature is effectively zero and relaxation results from the local yielding of contacts triggered by externally applied vibrations.

We now turn to a tentative microscopic model to capture this slowing-down process. We first need to connect

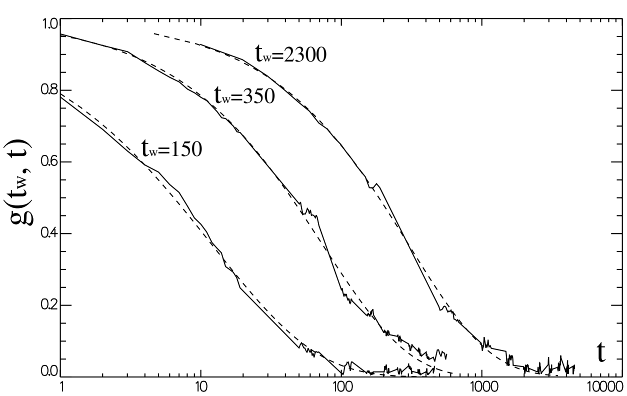

FIG. 2. Three different correlation functions obtained after three different times t_w. Solid lines correspond to experimental values and dashed lines to the stretched exponential fit.

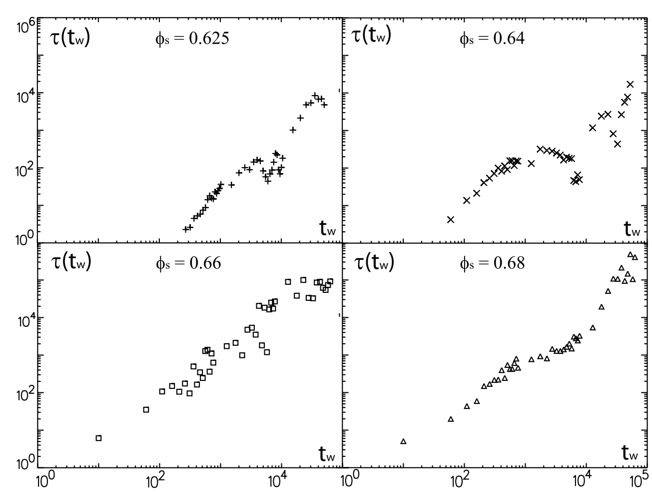

FIG. 3. Evolution of the dynamical time τ with the number of low magnitude taps t_w for different packing fractions ϕ_s.

DWS measurements to a grain-level description of the dynamics. We note that only the irreversible grain displacements produced by slippage events are responsible for the speckle image decorrelation. Since we do not observe significant compaction in the low vibration regime, the amplitude of these displacements ought to be much smaller than the diameter of the grains. Assuming that they are uniformly distributed in space and have a unique characteristic amplitude, $\tau(t_w)$ is simply proportional to the inverse of the yielding frequency [17,23].

The observed dynamical arrest is rather insensitive to the packing fraction of the sample. Any satisfactory description thus requires the introduction of another internal variable that will control the instantaneous response of the pack to gentle vibrations. Here we propose to focus on contact stress distribution. It has been observed that the form of this distribution is almost independent of the volume fraction of the pack and the preparation history [1–3]. However, standard measurements are not sensitive enough to detect small variations in these distributions that may follow from very gentle mechanical vibrations. We will argue here that the observed evolution of the dynamics follows from a slow modification of the stress distribution which effectively strengthens the granular pile.

To model such a dynamics, we picture the granular assembly as a set of independent contacts (whose total number is supposed to be a constant.) Each contact is characterized at time t by the normal and tangential component of the contact force which we denote σ_n and σ_t, respectively. Mechanical equilibrium imposes that $\sigma_t < \mu\sigma_n$. For simplicity, we will make the friction coefficient μ equal to 1 in the rest of the Letter. For a given packing structure, the state of the internal stress field is characterized by the two variables stress density distribution $P_\sigma(\sigma_n, \sigma_t)$. As the cell is vibrated, mechanical waves traveling through the sample induce random stress fluctuations at each contact. Such perturbations can locally trigger the rupture of a contact, whenever the shear force σ_t overcomes the normal force σ_n. We assume an exponential distribution $\chi(\delta\sigma)$ of the maximum force fluctuation induced by the tap on each contact:

$$\chi(\delta\sigma) = \frac{1}{\overline{\delta\sigma}}\exp\left(-\frac{\delta\sigma}{\overline{\delta\sigma}}\right). \tag{2}$$

In this expression, the mean force fluctuation $\overline{\delta\sigma}$ is an increasing function of the applied vibration amplitude. Thus the probability for a given contact to yield following a single tap writes

$$\omega_y(\sigma_n, \sigma_t) = \exp\left(-\frac{\sigma_n - \sigma_t}{\overline{\delta\sigma}}\right). \tag{3}$$

This expression is a consequence of the peculiar form [Eq. (2)] taken for the tap induced force fluctuations $\chi(\delta\sigma)$. However, the main results of the present model remain valid for any fast decaying distribution (faster than a power law).

After a yielding event, the force at the renewed contact is chosen from a given "rejuvenated" distribution which we consider intrinsic to the system. The distribution of normal forces in a granular pile under a moderate load is known to exhibit an exponential tail at high forces and a plateau below the mean force [1–3]. Numerical measurements have also shown that, for a given value of the normal force, the tangential forces are uniformly distributed between 0 and the sliding limit $\sigma_t = \mu\sigma_n$ (in a 2D case) [21]. We use these different observations to infer the form of the rejuvenated distribution $P_{\text{rej}}(\sigma_n, \sigma_t)$, which thus writes

$$P_{\text{rej}}(\sigma_n, \sigma_t) = \frac{1}{\sigma_0 \cdot \sigma_n}\exp\left(-\frac{\sigma_n}{\sigma_0}\right), \tag{4}$$

where σ_0 is the mean stress inside the pile. For simplicity, we have omitted the plateau saturation of the distribution at low forces. We can now derive the dynamical equation of evolution of the stress distribution P_σ:

$$\frac{\partial P_\sigma(\sigma_n, \sigma_t)}{\partial t} = -P_\sigma(\sigma_n, \sigma_t)\omega_y(\sigma_n, \sigma_t) + P_{\text{rej}}(\sigma_n, \sigma_t)F(P_\sigma), \tag{5}$$

where $F(P_\sigma)$ is the total frequency of sliding events which is self-consistently defined as

$$F(P_\sigma) = \iint_{\sigma'_t < \sigma'_n} P_\sigma(\sigma'_n, \sigma'_t)\omega_y(\sigma'_n, \sigma'_t)d\sigma'_n d\sigma'_t. \tag{6}$$

The present description exhibits many common features with Bouchaud's trap model [22] of glass transition. In the latter, the internal dynamics of a glassy liquid is pictured as a succession of thermal escapes from energy wells of various depths. In an analogous way, each contact here can be considered as frozen in a mechanical trap (the local solid friction cone), the depth of which depends on the relative amplitude of the normal and shear components of the contact force. Moreover, in the absence of temperature, mechanical vibrations play the role of the energy source by allowing individual contacts to hop out of their trap.

As in the trap model, we thus observe two limiting regimes depending on the relative values of the intensity of the applied stress $\overline{\delta\sigma}$ and the width σ_0 of the rejuvenated distribution. For large vibrations, i.e., $\overline{\delta\sigma} > \sigma_0$, the rejuvenated stress distribution is a stationary solution of Eq. (6), and the yielding frequency is constant in time. In contrast, for $\overline{\delta\sigma} < \sigma_0$, the stress distribution evolves endlessly. Figure 4 shows the time evolution of P_σ obtained by numerically solving Eq. (6) for $\overline{\delta\sigma} = \sigma_0/20$, starting with $P_\sigma = P_{\text{rej}}$. It shows that fragile contacts — contacts of low normal force or close to the sliding limit (inset) — are slowly depleted. As a result, the number of sliding events per time unit decays. More quantitatively,

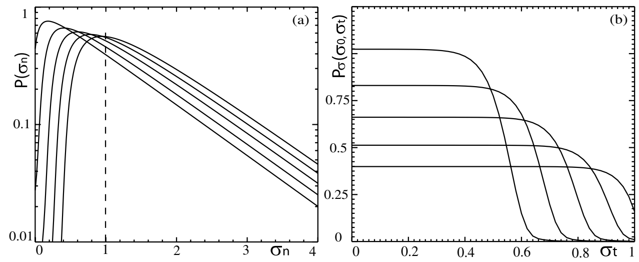

FIG. 4. Results of the numerical model for $\sigma_0 = 1$ and $\overline{\delta\sigma} = \sigma_0/20$: (a) distributions of normal forces for $t_w = 1$, 10, 100, 1000, and 10 000 (from left to right); (b) distributions of tangential forces for $\sigma_n = \sigma_0$ at the same times t_w (from bottom to top).

we find that the characteristic time between events grows linearly with the elapsed time, in reasonable agreement with our observations (Fig. 3.)

We have evidenced, through MSDWS measurements, the existence of a slowing down of the microscale dynamics over more than five decades in gently vibrated granular piles. This behavior is reminiscent of the aging process observed in glassy systems. This dynamics appears to be weakly connected to the overall grain-scale structure, which suggests a two-level description of granular systems. Under strong vibrations, a granular pile evolves through the restructuration of the piling geometry, leading to a slow irreversible compaction. In this regime, the forces network is rapidly renewed and shows no history-dependent behavior. Under gentle vibrations however, the geometry of the pile is essentially frozen but the forces network can still evolve by slowly depleting the most fragile contacts. This leads to an effective reinforcement of the pack structure as is evidenced in the present study by the decrease of vibration induced plastic events.

The precise nature of the yielding events remains however unclear. In particular, one might expect large spatial and temporal correlations between the events, which we cannot probe with DWS. Another important question concerns the relevance of such modifications to the onset of macroscopic flow. For instance, does the observed reinforcement of the force networks play a role in changing the threshold of avalanche triggering or shear-banding appearance?

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*Electronic address: georges.debregeas@college-de-france.fr

[1] D. M. Mueth, H. M. Jaeger, and S. R. Nagel, Phys. Rev. E **57**, 3164 (1998).
[2] G. Løvoll, K. J. Måløy, and E. G. Flekkøy, Phys. Rev. E **60**, 5872 (1999).
[3] D. L. Blair, N.W. Mueggenburg, A. H. Marshall, H. M. Jaeger, and S. R. Nagel, Phys. Rev. E **63**, 041304 (2001).
[4] L. Staron, J.-P. Vilotte, and F. Radjai, Phys. Rev. Lett. **89**, 204302 (2002).
[5] J. Desrues, R. Chambon, M. Mokni, and F. Mazerolle, Geotechnique **46**, 529 (1996).
[6] C. H. Liu, Phys. Rev. B **50**, 782 (1994).
[7] D. Howell, R. P. Behringer, and C. Veje, Phys. Rev. Lett. **82**, 5241 (1999).
[8] M. Da Silva and J. Rajchenbach, Nature (London) **406**, 708 (1999).
[9] J. B. Knight, C. G. Fandrich, C. N. Lau, H. M. Jaeger, and S. R. Nagel, Phys. Rev. E **51**, 3957 (1995).
[10] P. Philippe and D. Bideau, Europhys. Lett. **60**, 677 (2002).
[11] C. Lesaffre, V. Mineau, D. Picart, and H. Van Damme, C. R. Acad. Sci. Paris, Ser. IV **1**, 647 (2000).
[12] L. Cipelletti and D. A. Weitz, Rev. Sci. Instrum. **70**, 3214 (1999).
[13] V. Viasnoff, F. Lequeux, and D. Pine, Rev. Sci. Instrum. **73**, 2336 (2002).
[14] N. Menon and D. J. Durian, Science **275**, 1920 (1997).
[15] K. Kim, J. J. Park, J. K. Moon, H. K. Kim, and H. K. Pak, J. Korean Phys. Soc. **40**, 983 (2002).
[16] $l^\star$ was found equal to 220 ± 20 μm, which allows in our geometry the use of a diffusive model for the light in the sample [17].
[17] G. Maret and P. E. Wolf, Z. Phys. B **65**, 409 (1987); D. J. Pine, D. A. Weitz, P. M. Chaikin, and E. Herbolzheimer, Phys. Rev. Lett. **60**, 1134 (1988).
[18] A. Knaebel, M. Bellour, J.-P. Munch, V. Viasnoff, F. Lequeux, and J. L. Harden, Europhys. Lett. **52**, 73 (2000).
[19] V. Viasnoff and F. Lequeux, Phys. Rev. Lett. **89**, 065701 (2002).
[20] L. Cipelletti, L. Ramos, S. Manley, E. Pitard, D. A. Weitz, E. E. Pashkovski, and M. Johansson, Faraday Discuss. **123**, 237 (2003).
[21] F. Radjai, M. Jean, J.-J. Moreau, and S. Roux, Phys. Rev. Lett. **77**, 274 (1996).
[22] J.-P. Bouchaud, J. Phys. I (France) **2**, 1705 (1992).
[23] D. J. Durian, D. A. Weitz, and D. Pine, Science **252**, 686 (1991).